

Meeting 7

Math 22

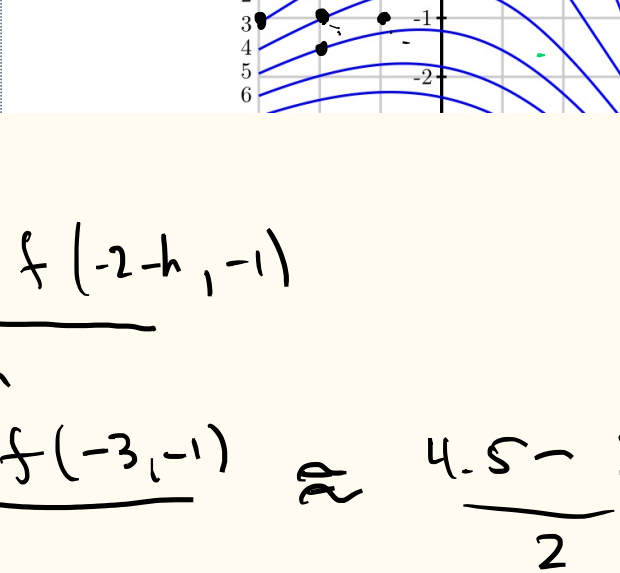
$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} \text{ for small } h$$

Activity 10.2.6

- Complete as a class

- Estimate the partial derivative $f_x(-2, -1)$. (Hint: How can you find values of f that are of the form $f(-2+h)$ and $f(-2-h)$ so that you can use a symmetric difference quotient?)
- Estimate the partial derivative $f_y(-2, -1)$.
- Estimate the partial derivatives $f_x(-1, 2)$ and $f_y(-1, 2)$.
- Locate, if possible, one point (x, y) where $f_x(x, y) = 0$.
- Locate, if possible, one point (x, y) where $f_x(x, y) < 0$.
- Locate, if possible, one point (x, y) where $f_x(x, y) > 0$.

Activity 10.2.6. Shown below in Figure 10.2.8 is a contour plot of a function f . The values of the function on a few of the contours are indicated to the left of the figure.



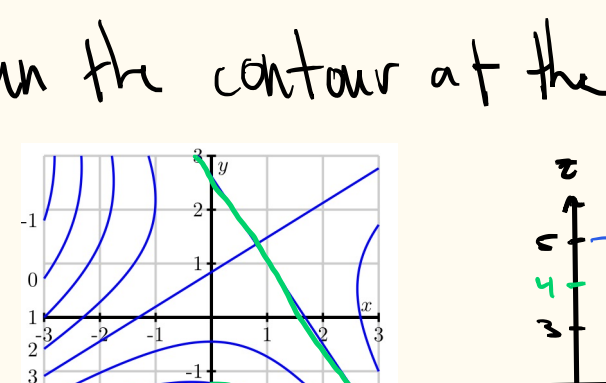
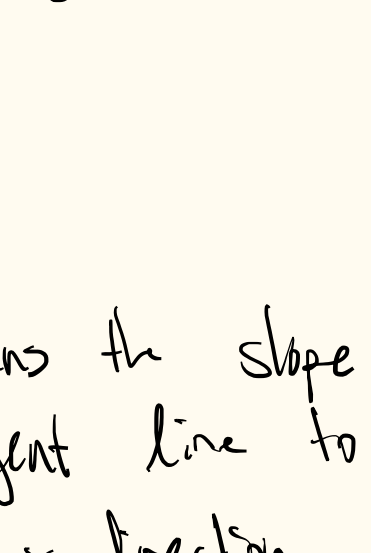
(a) $f_x(-2, -1) \approx \frac{f(-2+h, -1) - f(-2-h, -1)}{2h}$

$$\approx \frac{f(-1, -1) - f(-3, -1)}{2} \approx \frac{4.5 - 3}{2} \approx \frac{1.5}{2} = \frac{3}{4}$$

(b) $f_y(-2, -1) \approx \frac{f(-2, -1+h) - f(-2, -1-h)}{2h}$

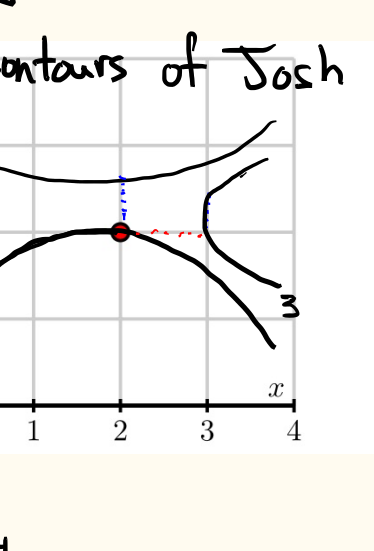
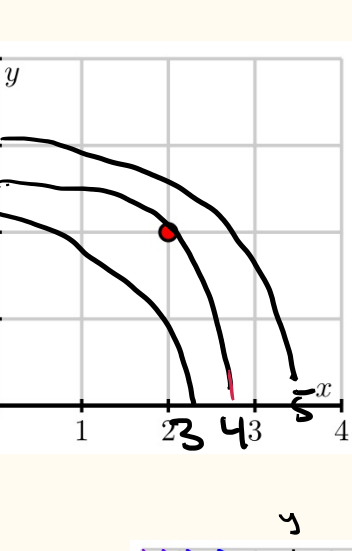
$$\approx \frac{f(-2, -0.5) - f(-2, -1.5)}{1} = -2$$

(c) $f_x(0, -5) \approx 0$ because the contours to the left and right are at equal elevations.



This means the slope of the tangent line to the graph at $(0, -5)$ in the x direction is zero. The other red dots also have the same property.

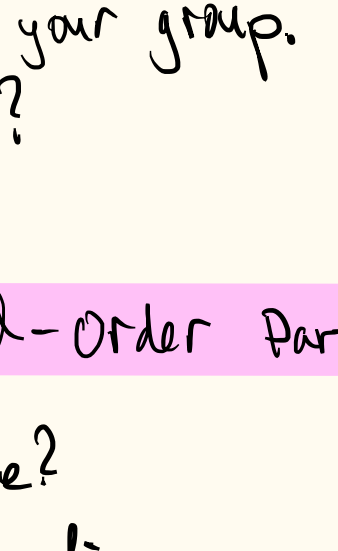
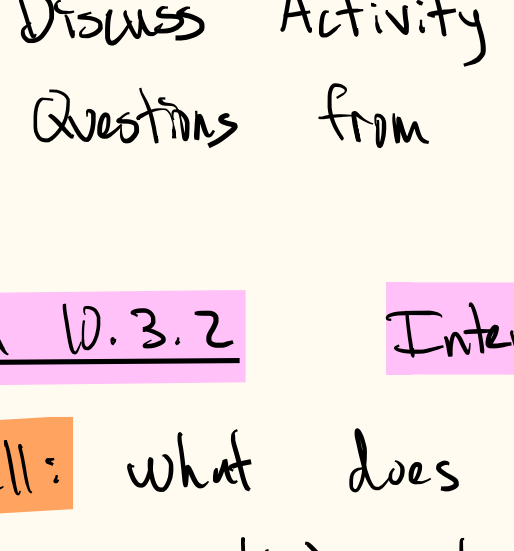
(e) $f_x(2, -1.5) < 0$ because the contour to the left is at a higher elevation than the contour at the right.



This means the slope of the tangent line to the graph at $(2, -1.5)$ in the x direction is negative. The other red dots also have the same property.

(f) No points on the graph satisfy $f_y(x, y) > 0$.

- Suppose you have a different function g , and you know that $g(2, 2) = 4$, $g_x(2, 2) > 0$, and $g_y(2, 2) > 0$. Using this information, sketch a possible contour for the function $g(x, y) = 4$ passing through $(2, 2)$ on the left side of Figure 10.2.9. Then include possible contours $g(x, y) = 3$ and $g(x, y) = 5$.
- Suppose you have yet another function h , and you know that $h(2, 2) = 4$, $h_x(2, 2) < 0$, and $h_y(2, 2) > 0$. Using this information, sketch a possible contour $h(x, y) = 4$ passing through $(2, 2)$ on the right side of Figure 10.2.9. Then include possible contours $h(x, y) = 3$ and $h(x, y) = 5$.



Section 10.3

Second-Order Partial Derivatives

Reading Debrief

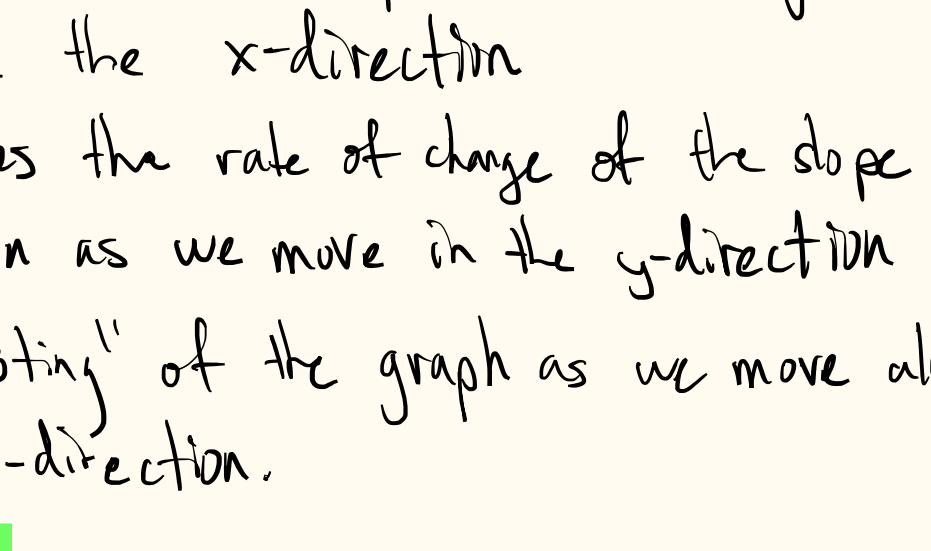
- Discuss Activity 10.3.2 w/ your group.
- Questions from Section 10.3.1?

Section 10.3.2

Interpreting Second-Order Partial Derivatives

Recall: what does $f''(x)$ measure?

- $f'(x)$ = slope of tangent line
- $f''(x)$ measures concavity of the graph of



$$f'(a) > 0 \quad f''(a) = 0 \quad f''(a) < 0$$

What does f_{xx} measure?

- f_{xx} measures the concavity of a trace in the x -direction.

What does f_{yy} measure?

- f_{yy} measures the concavity of a trace in the y -direction

What does f_{xy} measure?

- f_{xx} measures the slope of the tangent line to a trace in the x -direction
- f_{xy} measures the rate of change of the slope in the x -direction as we move in the y -direction. Measures the "twisting" of the graph as we move along a trace in the y -direction.

Activity 10.3.4

- Complete w/ your group.
- Class discussion.

From 10.2.5

$$w_x(20, -10) \approx -0.5 \quad w_y(20, -10) \approx 1.3$$

$v(T)$	-30	-25	-20	-15	-10	-5	0	5	10	15	20
5	-46	-40	-34	-28	-22	-16	-11	-5	1	7	13
10	-53	-47	-41	-35	-28	-22	-16	-10	-4	3	9
15	-58	-51	-45	-39	-32	-26	-19	-13	-7	0	6
20	-61	-55	-48	-42	-35	-29	-22	-15	-9	-2	4
25	-64	-58	-51	-44	-37	-31	-24	-17	-11	-4	3
30	-67	-60	-53	-46	-39	-33	-26	-19	-12	-5	1
35	-69	-62	-55	-48	-41	-34	-27	-21	-14	-7	0
40	-71	-64	-57	-50	-43	-36	-29	-22	-15	-8	-1

(a) $w_{TT}(20, -10) \approx \frac{w_T(20, -10+h) - w_T(20, -10-h)}{2h}$

$$\approx \frac{w_T(20, -5) - w_T(20, -15)}{10}$$

$$\approx \frac{1.3 - (-1.3)}{10} = 0$$

The formula requires $w_T(20, -5)$ and $w_T(20, -15)$, which we also have to estimate:

$$w_T(20, -5) \approx \frac{w(20, 0) - w(20, -10)}{10} \approx \frac{-22 - (-35)}{10} = 1.3\% / F$$

$$w_T(20, -15) \approx \frac{w(20, -10) - w(20, -20)}{10} = \frac{-35 - (-48)}{10} = 1.3\% / F$$

This tells us that $(20, -15)$ is an inflection point of $w(20, T)$. When the wind speed is 20mph, the relationship between wind chill and ambient temp is approximately linear. The rate of change of wind chill at a fixed wind speed does not change as temp changes.

Section 10.4

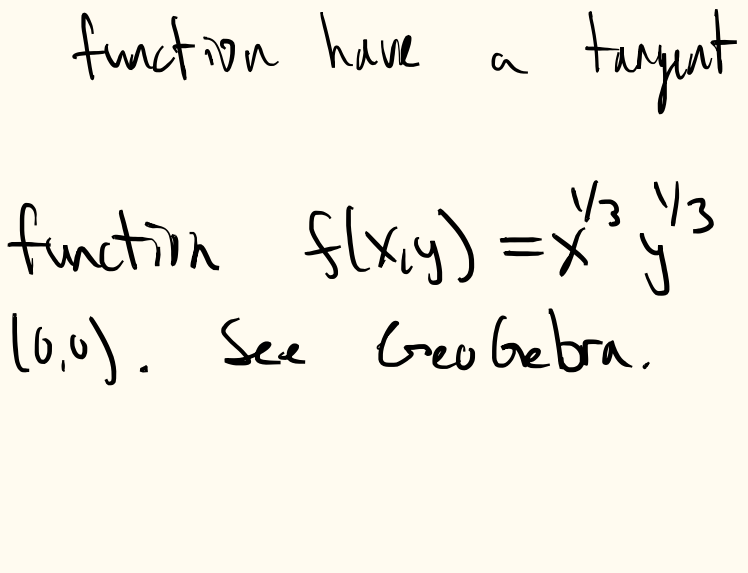
Tangent Planes and Differentials

Reading Debrief

- Discuss Activity 10.4.2 w/ your group.
- Questions from Section 10.4.1?

Q: Where does the tangent plane formula come from?

Suppose the plane $z = z_0 + a(x - x_0) + b(y - y_0)$ is tangent to the graph of f at (x_0, y_0) .



What is $\frac{\partial L}{\partial x}(x_0, y_0)$? If the graph is "locally linear", then the picture suggests:

$$\frac{\partial L}{\partial x}(x_0, y_0) = f_x(x_0, y_0)$$

From the formula for $L(x, y)$:

$$\frac{\partial L}{\partial x}(x_0, y_0) = a$$

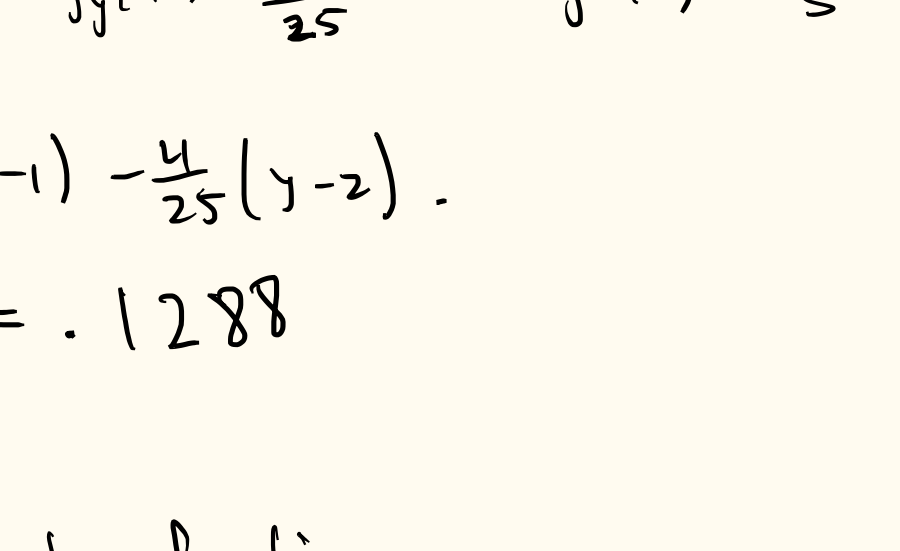
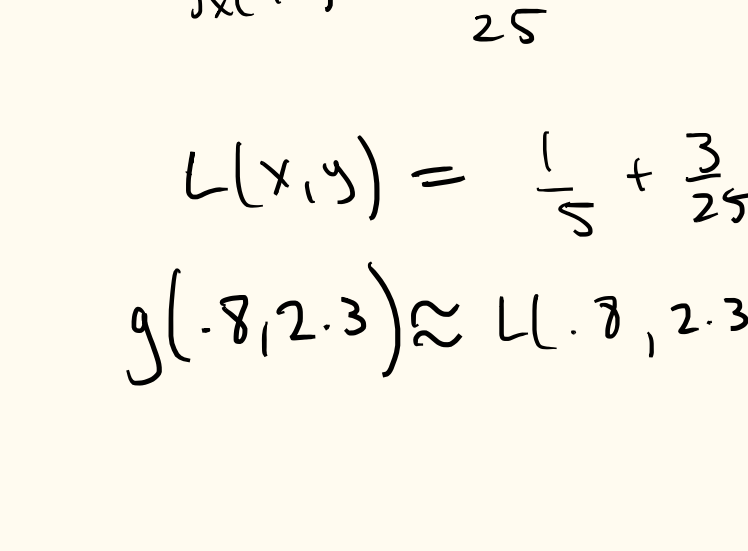
So $f_x(x_0, y_0) = a$. Similarly, $f_y(x_0, y_0) = b$.

Tangent Plane If $f(x, y)$ is continuously diff. at (x_0, y_0) , then the tangent plane to the graph of f at (x_0, y_0) exists and has scalar equation:

$$z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Another answer to the Q:

The tangent plane to f at (x_0, y_0) contains the tangent lines to the $x = x_0$ and $y = y_0$ traces.



A vector parallel to $x = x_0$ trace tangent line at $y = y_0$ is $v = (0, 1, f_y(x_0, y_0))$. A vector parallel to $y = y_0$ trace at $x = x_0$ is $u = (1, 0, f_x(x_0, y_0))$. A normal vector for tangent plane is

$$n = u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & f_y \\ 1 & 0 & f_x \end{vmatrix} = \langle f_x(x_0, y_0), f_y(x_0, y_0), -1 \rangle$$

The equation of this line is

$$0 = n \cdot (x - x_0, y - y_0, z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + z_0 - z$$

Q: What information about f can be read from the tangent plane at (x_0, y_0) .

A: You can read the values $f_x(x_0, y_0)$, $f_y(x_0, y_0)$, $f(x_0, y_0)$.

Q: Suppose $2x - 3y + 4z = 7$ is tangent to f at $(1, -1)$.

What are $f(1, -1) = 1$

$$L(x, y) = z = \frac{7}{4} - x + \frac{3}{4}y$$

A: $f_x(1, -1) = -1$

$$f_y(1, -1) = \frac{3}{4}$$

Q: Does every function have a tangent plane at every point?

A: No. The function $f(x, y) = x^{1/3}y^{1/3}$ has no tangent plane at $(0, 0)$. See GeoGebra.

Section 10.4.2

Linearization

The linearization of a function $f(x, y)$ at a point (x_0, y_0) is the function $L(x, y)$ whose graph is the tangent plane to f at (x_0, y_0) :

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

The function $L(x, y)$ provides a "good" approximation of f when (x, y) is near (x_0, y_0) .

Activity 10.4.3

- Complete Activity 10.4.3 and discuss w/ your group.
- Class discussion.

Activity 10.4.3. In what follows, we find the linearization of several different functions that are given in algebraic, tabular, or graphical form.

- Find the linearization $L(x, y)$ for the function g defined by $g(x, y) = \frac{x}{x^2 + y^2}$ at the point $(1, 2)$. Then use the linearization to estimate the value of $g(0.8, 2.3)$.

b. Table 10.4.8 provides a collection of values of the wind chill $w(v, T)$, in degrees Fahrenheit, as a function of wind speed, in miles per hour, and temperature, also in degrees Fahrenheit.

$v(T)$	-30	-25	-20	-15	-10	-5	0	5	10	15	20
5	-46	-40	-34	-28	-22	-16	-11	-5	1	7	13
10	-53	-47	-41	-35	-28	-22	-16	-10	-4	3	9
15	-58	-51	-45	-39	-32	-26	-19	-13	-7	0	6
20	-61	-55	-48	-42	-35	-29	-22	-15	-9	-2	4
25	-64	-58	-51	-44	-37	-31	-24	-17	-11	-4	3
30	-67	-60	-53	-46	-39	-33	-26	-19	-12	-5	1
35	-69	-62	-55	-48	-41	-34	-27	-21	-14	-7	0
40	-71	-64	-57	-50	-43	-36	-29	-22	-15	-8	-1

Use the data to first estimate the appropriate partial derivatives, and then find the linearization $L(v, T)$ at the point $(20, -10)$. Finally, use the linearization to estimate $w(10, -10)$, $w(20, -12)$, and $w(18, -12)$. Compare your results to what you obtained in Activity 10.2.5

(a) $g_x = \frac{(x^2 + y^2) - x(2x)}{(x^2 + y^2)^2}$

$$= \frac{y^2 - x^2}{(x^2 + y^2)^2}$$

$$g_y = \frac{-2xy}{(x^2 + y^2)^2}$$

$$g_x(1, 2) = \frac{3}{25}$$

$$g_y(1, 2) = \frac{-4}{25}$$

$$g(1, 2) = \frac{1}{5}$$

$$L(x, y) = \frac{1}{5} + \frac{3}{25}(x - 1) - \frac{4}{25}(y - 2)$$

$$g(0.8, 2.3) \approx L(0.8, 2.3) = .1288$$

Out of time.

c. Figure 10.4.9 gives a contour plot of a continuously differentiable function f .

Figure 10.4.9. A contour plot of $f(x, y)$. After estimating appropriate partial derivatives, determine the linearization $L(x, y)$ at the point $(2, 1)$, and use it to estimate $f(2.2, 1)$, $f(2, 0.8)$, and $f(2.2, 0.8)$.

Figure 10.4.9. A contour plot of $f(x, y)$. After estimating appropriate partial derivatives, determine the linearization $L(x, y)$ at the point $(2, 1)$, and use it to estimate $f(2.2, 1)$, $f(2, 0.8)$, and $f(2.2, 0.8)$.

Figure 10.4.9. A contour plot of $f(x, y)$. After estimating appropriate partial derivatives, determine the linearization $L(x, y)$ at the point $(2, 1)$, and use it to estimate $f(2.2, 1)$, $f(2, 0.8)$, and $f(2.2, 0.8)$.

Figure 10.4.9. A contour plot of $f(x, y)$. After estimating appropriate partial derivatives, determine the linearization $L(x, y)$ at the point $(2, 1)$, and use it to estimate $f(2.2, 1)$, $f(2, 0.8)$, and $f(2.2, 0.8)$.

Figure 10.4.9. A contour plot of $f(x, y)$. After estimating appropriate partial derivatives, determine the linearization $L(x, y)$ at the point $(2, 1)$, and use it to estimate $f(2.2, 1)$, $f(2, 0.8)$, and $f(2.2, 0.8)$.